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Estimated Standard Error of a Mean When There are only two Observations

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Theorem: When $n = 2$, the estimated standard error of the mean is

$$\hat{\sigma}_{\overline{X}} = \frac{|X_1 - X_2|}{2}.$$

Proof: The well-know formula for the estimated standard error of a mean is

$$\hat{\sigma}_{\overline{X}} = \frac{\hat{\sigma}_X}{\sqrt{n}}. \tag{1}$$

When $n = 2$, an unbiased estimate of the variance of the individual scores is:

$$\begin{aligned}
\hat{\sigma}^2_X &= \sum_{i=1}^{2}(X_i - \overline{X})^2 \\
&= (X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 \\
&= \left(X_1 - \frac{X_1 + X_2}{2}\right)^2 + \left(X_2 - \frac{X_1 + X_2}{2}\right)^2 \\
&= \left(\frac{X_1 - X_2}{2}\right)^2 + \left(\frac{X_2 - X_1}{2}\right)^2 \\
&= 2\left(\frac{X_1 - X_2}{2}\right)^2 \\
&= \frac{(X_1 - X_2)^2}{2},
\end{aligned}$$

which means that the estimated standard deviation of the two scores is

$$\hat{\sigma}_X = \frac{|X_1 - X_2|}{\sqrt{2}}. \tag{2}$$

Replacing Equation 2 in Equation 1 gives

$$\hat{\sigma}_{\overline{X}} = \frac{|X_1 - X_2|}{\sqrt{2}\sqrt{2}} = \frac{|X_1 - X_2|}{2}.$$

QED

Lemma: Under normality assumptions, when $n = 2$, a 68% confidence interval for the mean is

- (X_2, X_1) if $X_1 > X_2$ or
- (X_1, X_2) if $X_1 < X_2$.

Proof: If $X_1 > X_2$, then a 68% confidence interval for the mean extends from

$$\overline{X} - \hat{\sigma}_{\overline{X}} = \overline{X} - \frac{X_1 - X_2}{2} = \frac{X_1 + X_2}{2} - \frac{X_1 - X_2}{2} = X_2$$

to

$$\overline{X} + \hat{\sigma}_{\overline{X}} = \overline{X} + \frac{X_1 - X_2}{2} = \frac{X_1 + X_2}{2} + \frac{X_1 - X_2}{2} = X_1.$$

The proof of the second part of the lemma is similar.

QED

Note: These proofs were originally provided by the author for use in the Achievement Levels Settings projects conducted by ACT, Inc. in the 1990s (see Reckase, 2000). In that context, the two scores, X_1 and X_2, were themselves means (over items and raters) for two groups of raters each of which independently evaluated half of the items in a standard setting process, and the resulting estimated standard error was for the mean of these means.

References

Reckase, M. D. (2000). *The evolution of the NAEP Achievement Levels Setting Process: A summary of the research and development efforts conducted by ACT.* Iowa City, IA: Act, Inc.