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Standard Errors of Equating for Equipercentile Equating with Log-Linear Pre-Smoothing using the Delta Method*

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Abstract

Holland, King, and Thayer (1989) derived a formula for analytical standard error of equating (SEE) using the δ-method for the kernel equating method (see also Holland & Thayer, 1989; and von Davier, Holland & Thayer, 2004). Extending their derivation, this paper derives an analytical SEE procedure for the conventional percentile rank-based equipercentile equating with log-linear smoothing. This procedure is illustrated with real test data and is evaluated using both the bootstrap and parametric bootstrap (simulation) methods for three different equating designs.

1 Introduction

Equating errors can be decomposed into two parts: systematic equating errors and random equating errors. Systematic errors are usually caused by assumption failures in the equating method, bias in the sample statistics, etc. Random equating errors are caused by sampling errors. The standard error of equating (SEE) is an index for random equating error. It is defined as the standard deviation of the equated scores over repeated samples. Kolen and Brennan (2004, pp. 231-265) provides a detailed descriptions of available methods for estimating SEE's for various equating designs and methods. The bootstrap method can be applied to virtually all equating designs and methods. Analytical SEE estimation procedures based on the δ-method are available for linear methods and equipercentile methods without smoothing under various designs. Notably missing are analytical procedures for equipercentile equating with smoothing. The complexity of the smoothing procedures has added much difficulty to deriving the analytic SEE's for equipercentile equating with smoothing using the δ-method. Liou and Cheng (1997) derived analytical SEE's for various equating designs and methods using a non-δ-method-based approach. But their procedure is quite complicated to implement when log-linear smoothing is applied.

Interestingly, δ-method-based analytical SEE procedures have been derived for kernel equating methods (Holland & Thayer, 1989; Holland, King, & Thayer, 1989; von Davier, Holland, & Thayer, 2004). The kernel equating method involves a log-linear smoothing step. The difference between conventional equipercentile equating with pre-smoothing and the kernel method lies only at the continuization step. The purpose of this paper is to demonstrate that the analytical SEE derivations for kernel equating can be extended to conventional equipercentile equating with log-linear pre-smoothing. Because the δ-method isolates the different steps in the equating process into different components in deriving the SEE, it is easy to extend the SEE results from one equating method to another as long as both methods share some common steps.

2 The SEE for Kernel Equating

Holland and Thayer (1989) proposed a Gaussian kernel equating method. von Davier, Holland and Thayer (2004) re-presented this category of methods under a unified five-step framework (ch. 3. pp. 45-47) of test equating which can be applied to various data collection designs and equating methods. In this framework, the first step is log-linear smoothing applied to the raw frequency data. The step for estimating the marginal score distributions for the common synthetic population from the smoothed score distributions is capsulated in a concept called the Design Function. For the equivalent groups (EG) design, the Design Function is simply an identity function. For other designs, the implementation of the Design Function can be quite messy, especially for the non-equivalent groups anchor test (NEAT) design (see von Davier et al., 2004, chapter 2). The advantage of introducing this Design Function is it modularizes the equating process so that equating methods for different designs can be expressed in a unified framework.

Holland, King, and Thayer (1989) derived analytical SEE estimation procedures for the kernel equating methods using the δ-method, which are again presented in von Davier et al. (Equation 5.15). Their derivation "divides the problem in three" and ends up with an expression with three components: (For convenience, I will use the same terminology and notations as in von Davier et al. throughout this paper).

$$SEE_Y(x) = \|J_{e_Y} J_{DF} C\|, \tag{1}$$

where $\|v\| = \sqrt{\sum_j v_j^2}$ is the Euclidian norm of the vector v. The three components on the right hand side of the above equation are related to the three different steps in the kernel equating method. C is a matrix that characterizes the variance-covariance matrix of estimates of score distributions from the log-linear model, $\hat{R}$ and $\hat{S}$, by the following relationship (von Davier et al., Equation 5.11):

$$\Sigma_{\hat{R},\hat{S}} = CC^t, \tag{2}$$

where C^t means the transpose of C. The procedure for computing the C matrix is described in von Davier et al. (p. 52) and will not be repeated here.

J_{DF} is a matrix which is the component that relates to the Design Function. J_{DF} can be expressed as (von Davier et al., Equation 5.10):

$$J_{DF} = \begin{pmatrix} \frac{\partial r}{\partial R} & \frac{\partial r}{\partial S} \\ \frac{\partial s}{\partial R} & \frac{\partial s}{\partial S} \end{pmatrix}, \tag{3}$$

where r and s are the marginal score probabilities for the new and old test forms on the target population. They are the results of applying the Design Function to the log-linear smoothed univariate or bivariate score distributions.

J_{e_Y} is a vector which is the component that relates to the equating step after r and s are estimated. It can be expressed as (von Davier et al., Equation 5.9):

$$J_{e_Y} = \left(\frac{\partial e_Y}{\partial r}, \frac{\partial e_Y}{\partial s} \right), \tag{4}$$

where e_Y is the equating function. With kernel equating, the discrete score probabilities r and s are continuized using the Gaussian kernel to produce two continous cdf's F and G. Given F and G, e_Y defined as

$$e_Y(x; r, s) = G^{-1}(F(x; r); s). \tag{5}$$

The derivatives in Equation 4 can be expressed as (von Davier et al., Equation 5.16, 5.17):

$$\frac{\partial e_Y}{\partial r_j} = \frac{1}{G'} \frac{\partial F(x; r)}{\partial r_j}, \tag{6}$$

$$\frac{\partial e_Y}{\partial s_k} = -\frac{1}{G'} \frac{\partial G(e_Y(x); s)}{\partial s_k}, \tag{7}$$

where

$$G' = \frac{\partial G(e_Y(x); s)}{\partial y}. \tag{8}$$

The expressions for $\frac{\partial F(x;r)}{\partial r_j}$ and $\frac{\partial G(e_Y(x);s)}{\partial y}$ can be found in von Davier et al. (Equations 5.21, 5.22).

3 The SEE for Equipercentile Equating with Log-Linear Pre-smoothing

As mentioned previously, The difference between the conventional equipercentile equating with pre-smoothing and the kernel method lies only at the continuization step. While the kernel method applies the Gaussian kernel to marginal score distributions on the target population r and s to produce the continuous cdf's F and G, the conventional equipercentile method computes the percentile rank (PR) functions P and Q and then defines the equating function as

$$e_Y(x; r, s) = Q^{-1}(P(x; r); s), \tag{9}$$

which is parallel to Equation 5 except that P and Q are used in place of F and G. (For this reason, the conventional equipencentile method will be called PR-based equipercentile method throughout the rest of this paper.) So the derivation of the SEE for PR-based equipercentile equating with log-linear pre-smoothing should be parallel to the derivation of the SEE for kernel equating. The resulting SEE expression will be the same as Equation 1 except that one of the three components of SEE, J_{e_Y}, will have different expressions. The elements in right hand side of Equation 4 can be expressed as:

$$\frac{\partial e_Y}{\partial r_j} = \frac{1}{Q'} \frac{\partial P(x; r)}{\partial r_j}, \tag{10}$$

$$\frac{\partial e_Y}{\partial s_k} = -\frac{1}{Q'} \frac{\partial Q(e_Y(x); s)}{\partial s_k}, \tag{11}$$

where

$$Q' = \frac{\partial Q(e_Y(x); s)}{\partial y}. \tag{12}$$

According to the definition of the percentile rank function in Kolen and Brennan (2004, p. 44, Equation 2.14), we can express P in terms of r and Q in terms of s. For any given possible noninteger score x, we define x^* as the integer that is closest to x such that $x^* - .5 \leq x < x^* + .5$. Similarly we can define y^* as the integer that is closest to y. Then, the percentile rank functions $P(x; r)$ and $Q(y; s)$ can be expressed as

$$P(x; r) = \sum_{j=1}^{x^*-1} r_j + [x - (x^* - .5)] r_{x^*}. \tag{13}$$

$$Q(y; s) = \sum_{k=1}^{y^*-1} s_k + [y - (y^* - .5)]s_{y^*}. \tag{14}$$

Subsequently the partial derivatives needed in Equations 10 through 12 can be expressed as

$$\frac{\partial P(x; r)}{\partial r_j} = \begin{cases} 1 & \text{if } j = 1, 2, \ldots, x^* - 1 \\ x - (x^* - .5) & \text{if } j = x^* \\ 0 & \text{if } j > x^*, \end{cases} \tag{15}$$

$$\frac{\partial Q(e_Y(x); s)}{\partial s_k} = \begin{cases} 1 & \text{if } k = 1, 2, \ldots, y^* - 1 \\ e_Y(x) - (y^* - .5) & \text{if } k = y^* \\ 0 & \text{if } k > y^*, \end{cases} \tag{16}$$

and

$$\frac{\partial Q(e_Y(x); s)}{\partial y} = s_{y^*}, \tag{17}$$

Now we have all the necessary elements to compute $SEE_Y(x)$ for the PR-based equipercentile equating with log-linear pre-smoothing.

4 Illustration and Evaluation Using Real and Simulated Data

In the following sections, we demonstrate and evaluate the above described procedure for obtaining the SEE for equipercentile equating with log-linear pre-smoothing under three different equating data collection designs: equivalent groups (EG) design (also called random groups design), counterbalanced (CB) design, and non-equivalent groups with anchor-test (NEAT) design (also called common-items non-equivalent group design) . Three data sets, each for a different data design, are used. Under the NEAT design, both the post-stratification method (also called frequency estimation method) and the chained equipercentile method are used.

4.1 Method

For each data set and equating method, the following steps are taken to carry out the analysis.

> *Step 1:* The log-linear smoothing method is applied first to obtain the smoothed score distribution. For the EG design, the univariate log-linear smoothing procedure is used, whereas for the CB and NEAT designs, the bivariate log-linear smoothing procedure is used.
>
> *Step 2.* The Design Function is applied to obtain the marginal score probability distribution for the target population. von Davier et al. (2004) contains detailed descriptions of the Design Functions for each of the corresponding designs.

Step 3. The equating function is computed using the PR-based equipercentile method (Kolen & Brennan, 2004). For comparison purposes, the kernel equating (von Davier et al. 2004) is also computed.

Step 4. Analytical SEE's are computed for both the PR-based equipercentile method and the kernel method.

Step 5. For comparison purposes, a bootstrap procedure (see Kolen & Brennan, 2004, chap. 7) is also used to compute the SEE for PR-based equipercentile equating (with log-linear smoothing), with 500 replications.

Step 6. A parametric bootstrap procedure (see Kolen & Brennan, 2004, chap. 7, also called parametric bootstrap) is used to evaluate the analytical SEE's. The smoothed distribution from Step 1 is used as the population distribution, then random samples with the same sample sizes are drawn from the population. The SEE's are computed based on 500 replications.

4.2 An Example for the Equivalent Groups (EG) Design

The data set for the equivalent groups design is taken from von Davier et al. (2004, chap. 7). In this data set, both the X and Y forms have 20 items. The sample size for form X is 1453, and that for form Y is 1455. The same degrees of log-linear smoothing (2 for X and 3 for Y) that were used in von Davier et al. are used here. The equating results and SEE's are contained in Table 1. The δ-method based SEE, the bootstrap SEE, and the parametric bootstrap-based SEE are plotted in Figure 1 for visual comparison.

It can be seen from Table 1 that the kernel method and the PR-based equipercentile method produced very similar equating functions. Their analytical SEE's are also very close to each other. Figure 1 shows that the analytical SEE's for the PR-based equipercentile method are close to the bootstrap and parametric bootstrap SEE's. Using parametric bootstrap SEE's as the criterion, the bootstrap performs slightly better than the analytical procedure except for the score range of 10 to 17.

4.3 An Example for the Counterbalanced (CB) Design

The data set for the counterbalanced design is taken from von Davier et al. (2004, chap. 9). For this data set, form X has 75 items, and form Y has 76 items. The group taking form X first has 143 examinees, and the group taking form Y first has 140 examinees. The same degrees of the bivariate log-linear smoothing (2 for X and Y marginals, 1 for cross product, for both groups) that were used in von Davier et al. are used here. The equating functions and SEE's are in Table 2. The SEE's for the PR-based equipercentile method are plotted in Figure 2 for visual comparison.

Figure 2 shows that the analytical SEE's are close to the parametric bootstrap and bootstrap SEE's. The analytical SEE's display some bumps in the lower half of the score scale. A closer look at the computational process shows that the bumps are caused by a combination of the piece-wise linear nature of the percentile rank function and the particular pattern in the equated scores. The quantity in Equation 17 is in the numerator in computing SEE. This quantity stays constant from half point below an integer score to half point above the integer score, and may have a quite large impact if the equated score is, say, 16.49 rather than 16.51. For example, in Table 2, at score point 10, the PR-based equipercentile equated score is 15.56. For next score point 11, the equated score is 16.42, which results in a sudden increase in SEE. Had the equated score been, say 16.51 rather 16.42, then SEE would not have a sudden increase. Except for the bumps, the analytical SEE's seem to be closer to the parametric bootstrap SEE's than the bootstrap SEE's.

4.4 An Example for the Post-Stratification Method under the Non-Equivalent Groups with Anchor Test (NEAT) Design

The data set for this design is taken from Kolen and Brennan (2004, chap. 4). Both form X and form Y have 36 items, with 12 common items. The sample size for form X is 1655, and for form Y is 1638. The degrees of the bivariate log-linear smoothings are the same for both groups: 3 for both X and Y marginals, and 1 for the cross-product. The equating functions and SEE's are in Table 3. The SEE's for the PR-based equipercentile method are plotted in Figure 3.

Table 3 again shows that the kernel method and the PR-based equipercentile method produced very similar equating functions. The analytical SEE's are very similar, also, except at the lower end of the score scale. Figure 3 shows that the analytical SEE's are very close to the bootstrap and parametric bootstrap SEE's at the middle of the score scale. Towards both ends of the scale, however, analytical SEE's are closer to the parametric bootstrap SEE's than the bootstrap SEE's.

4.5 An Example for the Chained Equipercentile Method under the Non-Equivalent Groups with Anchor Test (NEAT) Design

The same data set is used here as described in the previous subsection and the same log-linear model is used for smoothing. The equating functions and SEE's are in Table 4. The SEE's for the PR-based equipercentile method are plotted in Figure 4.

The patterns of comparisons here are very similar to those for the post-stratification method. Note that the chained equipercentile method produces consistently lower equated scores than the post-stratification method throughout most of the score scale.

5 Summary

This paper derives analytical SEE's using the δ-method for the PR-based equipercentile method with log-linear pre-smoothing. Much of the derivation builds upon the results in Holland et al. (1989) and von Davier et al. (2004). Three real data sets were used to illustrate results for the analytical SEE's compared to bootstrap SEE's and parametric bootstrap SEE's. The results show that the analytical SEE seem to be close to the parametric bootstrap results. In most cases, analytical SEE results seem slightly better than the bootstrap SEE results.

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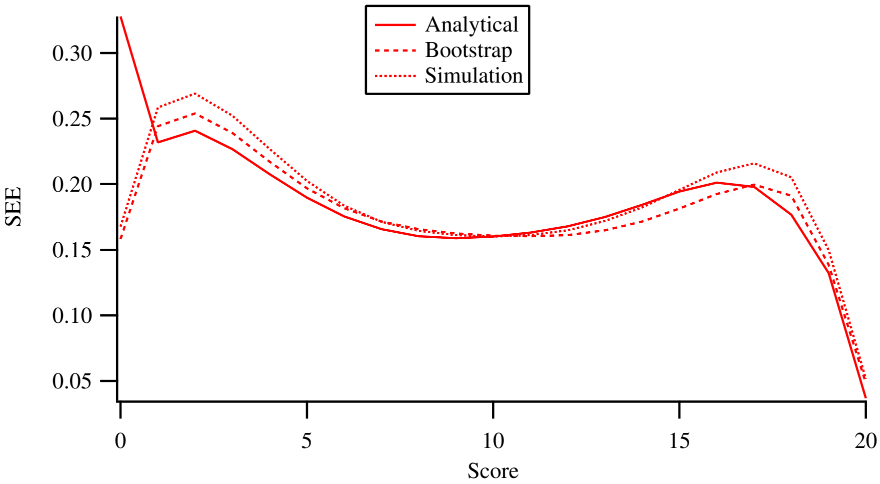

Figure 1: Analytical (δ-Method) and Bootstrap Standard Errors of Equating for the Equivalent Group (EG) Design

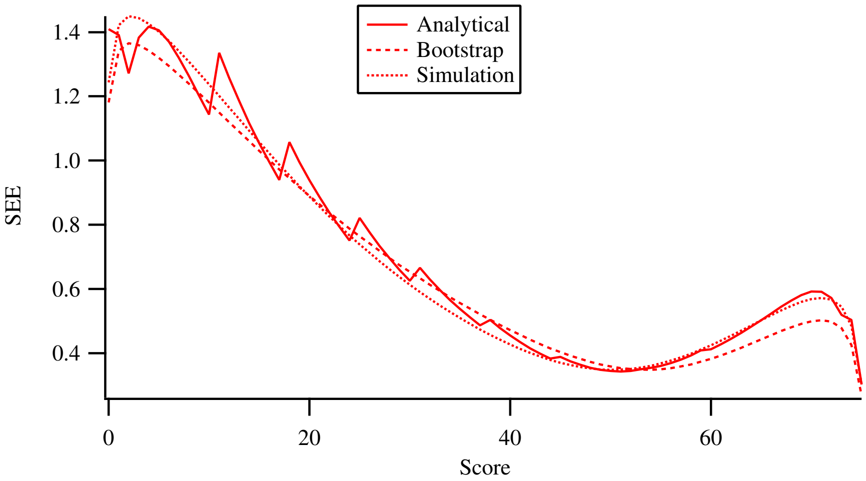

Figure 2: Analytical (δ-Method), Bootstrap and Parametric Bootstrap Standard Errors of Equating for the Counterbalanced (CB) Design

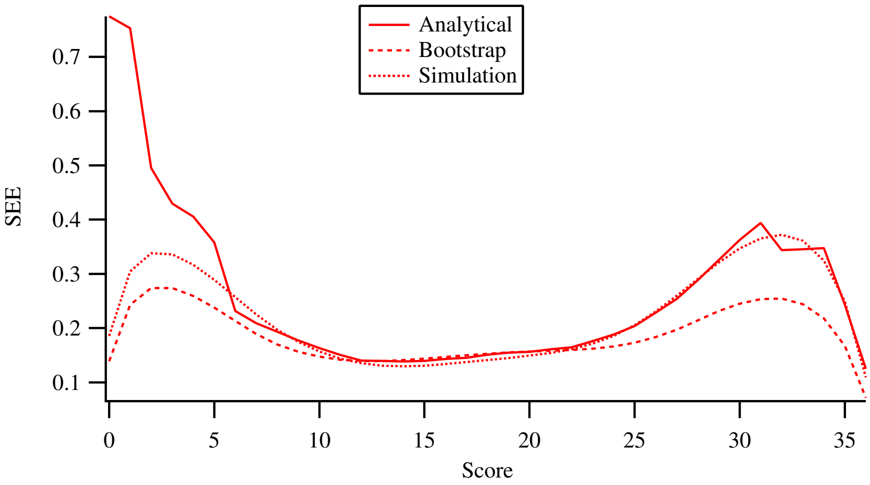

Figure 3: Analytical (δ-Method), Bootstrap and Parametric Bootstrap Standard Errors of Equating for the Post-Stratification Method under the Non-Equivalent Groups with Anchor Test (NEAT) Design

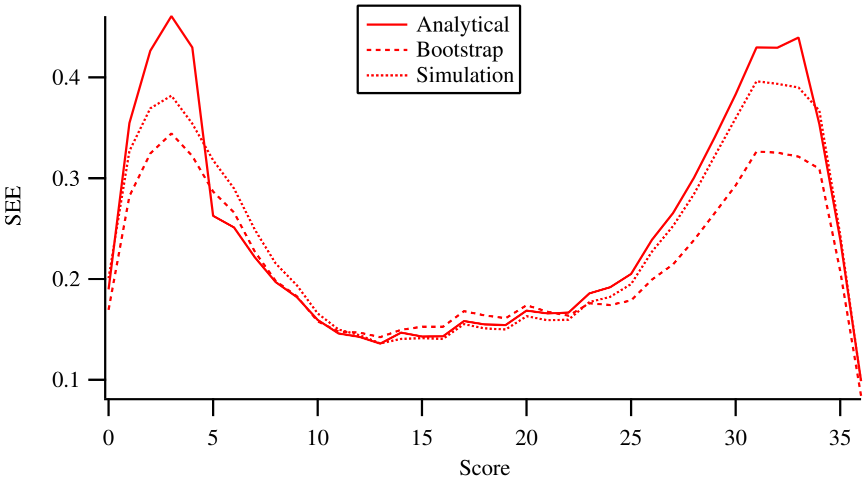

Figure 4: Analytical (δ-Method), Bootstrap and Parametric Bootstrap Standard Errors of Equating for Chained Equipercentile Method under the Non-Equivalent Groups with Anchor Test (NEAT) Design

Table 1: Equating Functions and Standard Errors of Equating for the Equivalent Groups (EG) Design

Score	Kernel Eq.	Kernel SEE	Log-lin. Eq.	Log-lin. SEE	Bootst. SEE	Param. SEE
0	0.394287	0.220048	0.468292	0.327861	0.158266	0.167591
1	1.581268	0.289534	1.636636	0.231776	0.244248	0.258364
2	2.640305	0.287514	2.676183	0.240641	0.253886	0.269074
3	3.644409	0.266394	3.674334	0.226713	0.239047	0.252306
4	4.631634	0.241040	4.659078	0.207563	0.217276	0.226891
5	5.617771	0.216952	5.642678	0.189622	0.196960	0.202475
6	6.609974	0.196666	6.631269	0.175375	0.181732	0.183610
7	7.612019	0.181244	7.628504	0.165676	0.171669	0.171324
8	8.625965	0.170751	8.636953	0.160425	0.165783	0.164352
9	9.652987	0.164572	9.658548	0.158871	0.162516	0.161101
10	10.693469	0.161871	10.694570	0.160000	0.160796	0.160163
11	11.747141	0.162101	11.745358	0.163058	0.160253	0.161285
12	12.812614	0.165336	12.809784	0.167919	0.161263	0.164936
13	13.886886	0.172134	13.884516	0.174984	0.164771	0.171892
14	14.964132	0.182653	14.963003	0.184357	0.171554	0.182461
15	16.033897	0.195051	16.034101	0.194514	0.181479	0.195793
16	17.078114	0.203760	17.080085	0.201275	0.192541	0.209064
17	18.067684	0.199005	18.073613	0.197903	0.199709	0.215796
18	18.960724	0.169998	18.972764	0.176665	0.191270	0.205339
19	19.718323	0.118601	19.712560	0.132193	0.138493	0.149831
20	20.393066	0.070308	20.289607	0.036886	0.048534	0.051488

Table 2: Equating Functions and Standard Errors of Equating for the Chained Equipercentile Method under the Counterbalanced (CB) Design

Score	Kernel Eq.	Kernel SEE	Log-lin. Eq.	Log-lin. SEE	Bootst. SEE	Param. SEE
0	3.230469	1.291430	3.220647	1.409535	1.181308	1.243489
1	5.833984	1.443920	5.851437	1.390636	1.341910	1.421380
2	7.471680	1.468856	7.524331	1.271684	1.365238	1.449630
3	8.794434	1.455257	8.797906	1.383116	1.358785	1.443624
4	9.928223	1.436354	9.927047	1.416953	1.342152	1.425621
5	10.967529	1.411582	10.962527	1.405892	1.319805	1.401043
6	11.943848	1.383636	11.937183	1.369460	1.294472	1.372373
7	12.878174	1.353800	12.872472	1.319320	1.267208	1.340864
8	13.786255	1.322422	13.782531	1.262436	1.238914	1.307716
9	14.673340	1.290623	14.676737	1.202964	1.210179	1.273281
10	15.547302	1.257912	15.561327	1.143378	1.180273	1.237436
11	16.413391	1.223872	16.426676	1.335532	1.149683	1.201266
12	17.274231	1.188749	17.275886	1.257926	1.119739	1.165339
13	18.129822	1.153581	18.125657	1.185107	1.089815	1.129188
14	18.984100	1.118875	18.977208	1.117058	1.060272	1.093074
15	19.835754	1.085474	19.831227	1.053648	1.030369	1.057292
16	20.686096	1.053068	20.688047	0.994683	1.000924	1.022017
17	21.536438	1.020905	21.547764	0.939940	0.972387	0.987280
18	22.385468	0.988742	22.393301	1.057554	0.944610	0.953548
19	23.235153	0.956524	23.234363	0.995806	0.916877	0.920378
20	24.084839	0.924992	24.079984	0.938806	0.889618	0.887712
21	24.935181	0.894685	24.929799	0.886177	0.863414	0.856426
22	25.785522	0.865759	25.783434	0.837572	0.837845	0.825332
23	26.636192	0.837739	26.640508	0.792667	0.812478	0.795656
24	27.487190	0.810046	27.500647	0.751166	0.787826	0.766748
25	28.338516	0.782431	28.342668	0.820960	0.764018	0.738780
26	29.190826	0.755105	29.188965	0.775361	0.740907	0.712231
27	30.043301	0.728624	30.039197	0.733233	0.717950	0.685953
28	30.896431	0.703286	30.892942	0.694283	0.695402	0.660709
29	31.750053	0.679012	31.749792	0.658249	0.673844	0.636399
30	32.604168	0.655393	32.609357	0.624889	0.653363	0.612957
31	33.458611	0.632040	33.467772	0.666016	0.633105	0.590437
32	34.313709	0.608849	34.316148	0.629968	0.612940	0.568818
33	35.169464	0.586087	35.168015	0.596633	0.593418	0.547928
34	36.025875	0.564137	36.022963	0.565805	0.574560	0.528208
35	36.882778	0.543223	36.880599	0.537300	0.556259	0.509155
36	37.740173	0.523268	37.740547	0.510962	0.538267	0.490847
37	38.598061	0.503998	38.602442	0.486655	0.520813	0.473485
38	39.456440	0.485189	39.463062	0.503306	0.503734	0.456985
39	40.315475	0.466848	40.317450	0.477991	0.487585	0.441391
40	41.175003	0.449237	41.174485	0.454872	0.471982	0.426738
41	42.035269	0.432693	42.033795	0.433871	0.456742	0.412912
42	42.896109	0.417453	42.895021	0.414932	0.442035	0.400195
43	43.757441	0.403556	43.757817	0.398028	0.427926	0.388881
44	44.619224	0.390904	44.621846	0.383154	0.414823	0.378775
45	45.481458	0.379418	45.486151	0.387940	0.402880	0.370081
46	46.344225	0.369162	46.346158	0.374347	0.391860	0.362745
47	47.207607	0.360376	47.208059	0.363141	0.381743	0.356672
48	48.071646	0.353371	48.071519	0.354348	0.372526	0.352024
49	48.936258	0.348399	48.936216	0.347999	0.364560	0.348910
50	49.801363	0.345549	49.801834	0.344122	0.358069	0.347683
51	50.666878	0.344756	50.668070	0.342742	0.353185	0.348357
52	51.532885	0.345875	51.534631	0.343874	0.349972	0.350793
53	52.399425	0.348795	52.400649	0.349507	0.348305	0.354810
54	53.266621	0.353504	53.267605	0.353531	0.348197	0.360367
55	54.134596	0.360073	54.135665	0.359951	0.349749	0.367526
56	55.003391	0.368566	55.004629	0.368689	0.353059	0.376309
57	55.873130	0.378937	55.874350	0.379663	0.358120	0.386589
58	56.743935	0.390986	56.744750	0.392803	0.364778	0.398168
59	57.616093	0.404391	57.615852	0.408054	0.372820	0.410846
60	58.490137	0.418807	58.488181	0.412207	0.382062	0.424441
61	59.366888	0.433979	59.366198	0.427747	0.392436	0.438840
62	60.247248	0.449773	60.247761	0.444700	0.403892	0.453979
63	61.132569	0.466130	61.133921	0.462937	0.416275	0.469760
64	62.024616	0.482953	62.026218	0.482302	0.429366	0.485953
65	62.925644	0.500012	62.926854	0.502575	0.442659	0.501978
66	63.838892	0.516929	63.838950	0.523402	0.455893	0.517827
67	64.768789	0.533194	64.766920	0.544176	0.468784	0.533114
68	65.721077	0.548164	65.717024	0.563846	0.480773	0.547122
69	66.704285	0.561012	66.698198	0.580609	0.491171	0.559135
70	67.730305	0.570473	67.723291	0.591424	0.498968	0.568002
71	68.817345	0.574414	68.810927	0.591234	0.502382	0.571654
72	69.994110	0.569045	69.988320	0.571722	0.497594	0.566517
73	71.312435	0.547116	71.295580	0.519296	0.477920	0.544422
74	72.878448	0.484240	72.861694	0.502954	0.425437	0.484540
75	74.887512	0.303781	74.949741	0.301877	0.267872	0.303506

Table 3: Equating Functions and Standard Errors of Equating for the Post-Stratification Method under the Non-Equivalent Groups with Anchor Test (NEAT) Design

Score	Kernel Eq.	Kernel SEE	Log-lin. Eq.	Log-lin. SEE	Bootst. SEE	Param. SEE
0	0.031250	0.721241	0.021859	0.774931	0.170432	0.185385
1	1.082703	0.663308	1.078483	0.753121	0.288791	0.304445
2	2.155212	0.443328	2.158472	0.495561	0.325561	0.337848
3	3.237549	0.373099	3.249842	0.429462	0.326708	0.335868
4	4.323921	0.329852	4.346819	0.405658	0.310683	0.316833
5	5.412224	0.288354	5.445859	0.358087	0.286016	0.289130
6	6.500702	0.265777	6.532202	0.231648	0.257217	0.257282
7	7.588039	0.231205	7.606317	0.208788	0.227957	0.225376
8	8.672920	0.206887	8.682632	0.193407	0.201514	0.196936
9	9.754555	0.183896	9.759150	0.177354	0.179773	0.174308
10	10.832197	0.165608	10.834156	0.163227	0.163604	0.156879
11	11.905277	0.151467	11.906208	0.150395	0.151953	0.143864
12	12.973269	0.140637	12.974113	0.140205	0.143710	0.135126
13	14.035732	0.138305	14.036874	0.139611	0.138702	0.130509
14	15.092274	0.137087	15.093659	0.138349	0.136499	0.129400
15	16.142323	0.139043	16.143756	0.139644	0.136363	0.130797
16	17.185396	0.143041	17.186550	0.143349	0.137527	0.133663
17	18.220946	0.146366	18.221484	0.145258	0.139379	0.137247
18	19.248293	0.151592	19.248031	0.150776	0.141528	0.141054
19	20.266756	0.156318	20.265660	0.154532	0.143867	0.144957
20	21.275612	0.159795	21.273806	0.156109	0.146651	0.149152
21	22.274027	0.164979	22.271836	0.161084	0.150375	0.154211
22	23.261168	0.169875	23.259029	0.164711	0.155948	0.161113
23	24.236156	0.179771	24.234561	0.176212	0.164529	0.171125
24	25.198204	0.192186	25.197513	0.188240	0.177298	0.185486
25	26.146389	0.207542	26.146908	0.203794	0.195179	0.205137
26	27.080360	0.229743	27.081807	0.228227	0.218625	0.230238
27	27.999680	0.253541	28.001455	0.253958	0.246685	0.259623
28	28.904610	0.283045	28.905515	0.288236	0.276972	0.290611
29	29.796379	0.312959	29.794350	0.325316	0.306132	0.320659
30	30.676918	0.340418	30.669319	0.362587	0.330573	0.346709
31	31.549561	0.362222	31.533026	0.393993	0.347187	0.365022
32	32.419220	0.365552	32.406517	0.344021	0.352050	0.371873
33	33.292038	0.359378	33.285219	0.345436	0.339614	0.361114
34	34.176437	0.341482	34.172108	0.347336	0.301931	0.324440
35	35.083649	0.235827	35.078138	0.243067	0.228358	0.249041
36	36.032623	0.136908	36.018970	0.124644	0.096877	0.109075

Table 4: Equating Functions and Standard Errors of Equating for the Chained Equipercentile Method under the Non-Equivalent Groups with Anchor Test (NEAT) Design

Score	Kernel Eq.	Kernel SEE	Log-lin. Eq.	Log-lin. SEE	Bootst. SEE	Param. SEE
0	0.160400	0.204789	0.052595	0.189778	0.203030	0.200787
1	1.197815	0.309784	1.146772	0.354777	0.329077	0.327099
2	2.243652	0.351955	2.260515	0.426470	0.372489	0.369355
3	3.306335	0.357747	3.386299	0.460787	0.386504	0.381893
4	4.371475	0.340024	4.471103	0.429605	0.358044	0.354142
5	5.423279	0.311575	5.501137	0.262698	0.319854	0.317897
6	6.472099	0.280322	6.539170	0.251156	0.291021	0.289691
7	7.515656	0.247174	7.569780	0.221527	0.248062	0.248842
8	8.552544	0.217558	8.594404	0.197035	0.212571	0.215358
9	9.593380	0.191844	9.619451	0.181954	0.189637	0.193753
10	10.628996	0.170402	10.662153	0.159406	0.160494	0.165890
11	11.663383	0.155524	11.671584	0.145956	0.144064	0.150020
12	12.700359	0.145651	12.716576	0.142585	0.138505	0.144359
13	13.730601	0.140796	13.749858	0.136026	0.131100	0.136128
14	14.762159	0.140524	14.728883	0.146781	0.136500	0.140782
15	15.792643	0.142099	15.810096	0.142983	0.138447	0.141280
16	16.816589	0.145534	16.820450	0.143088	0.139318	0.140762
17	17.842203	0.149838	17.822538	0.158219	0.156010	0.155237
18	18.863518	0.153366	18.883752	0.154964	0.152994	0.150899
19	19.878844	0.157405	19.869716	0.154308	0.153528	0.149951
20	20.894478	0.161470	20.889197	0.168738	0.169736	0.163014
21	21.903421	0.165461	21.927967	0.165734	0.167234	0.159223
22	22.906311	0.171610	22.896913	0.166689	0.169648	0.159639
23	23.907051	0.180176	23.915729	0.185606	0.190016	0.176937
24	24.899017	0.192091	24.933067	0.191808	0.196878	0.182377
25	25.883526	0.209723	25.896551	0.205147	0.211109	0.194868
26	26.862595	0.232780	26.889483	0.238855	0.243952	0.227170
27	27.830959	0.260876	27.881543	0.265524	0.269956	0.252547
28	28.789673	0.294209	28.846894	0.300431	0.302748	0.284213
29	29.740578	0.329464	29.798658	0.340814	0.338538	0.322170
30	30.680252	0.362926	30.747485	0.383390	0.375879	0.359512
31	31.610626	0.390978	31.704866	0.429784	0.413503	0.396025
32	32.533806	0.404175	32.633167	0.429573	0.410935	0.393495
33	33.443649	0.392987	33.509377	0.439494	0.405736	0.389955
34	34.340508	0.348350	34.438392	0.353686	0.382929	0.367107
35	35.220169	0.252725	35.310383	0.239321	0.261726	0.243886
36	36.085266	0.133730	36.105634	0.098718	0.108737	0.098230